

# Reconstructing Dark Energy : A Comparison of Cosmological Parameters

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A large number of cosmological parameters have been suggested for obtaining information on the nature of dark energy. In this work, we study the efficacy of these different parameters in discriminating theoretical models of dark energy, using both currently available supernova (SNe) data, and simulations of future observations. We find that the current data does not put strong constraints on the nature of dark energy, irrespective of the cosmological parameter used. For future data, we find that the although deceleration parameter can accurately reconstruct some dark energy models, it is unable to discriminate between different models of dark energy, therefore limiting its usefulness. Physical parameters such as the equation of state of dark energy, or the dark energy density do a good job of both reconstruction and discrimination *if* the matter density is known to high accuracy. However, uncertainty in matter density reduces the efficacy of these parameters. A recently proposed parameter,  $\text{Om}$ , constructed from the first derivative of the SNe data, works very well in discriminating different theoretical models of dark energy, and has the added advantage of not being dependent on the value of matter density. Thus we find that a cosmological parameter constructed from the first derivative of the data, for which the theoretical models of dark energy are sufficiently distant from each other, and which is independent of the matter density, performs the best in reconstructing dark energy from SNe data.

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## I. INTRODUCTION

The nature of dark energy is one of the most tantalizing mysteries in cosmology today. Observations of high redshift type Ia supernovae tell us that the expansion of the universe is accelerating at present [1], which can not be satisfactorily explained in the standard cold dark matter (CDM) scenario. Independent observations of the cosmic microwave background [2] and large scale structure [3] tell us that two-thirds of the present density of the universe is composed of some unknown component. The study of this unknown “dark energy” is of great interest among cosmologists today.

Various theoretical models for dark energy have been suggested, the simplest being the cosmological constant model with constant dark energy density and equation of state  $w_{DE} = -1$ . Other models of dark energy include physically motivated models like the scalar field quintessence and Chaplygin gas models, as well as geometrically motivated models like scalar-tensor theories and higher dimensional braneworld models (see [4] and references therein).

The growing number of theoretical dark energy models has inspired a complementary, data-driven approach in which the properties of dark energy are reconstructed from the data by studying the cosmological parameters characterizing dark energy. Two primary methods are used for reconstructing cosmological parameters. In the first approach, known as parametric reconstruction, a sufficiently general fitting function is used to represent the parameter in the analysis. This suffers from the possibility of bias, depending on the form chosen for the parameter. The second method is that of non-parametric reconstruction, in which no specific form is assumed for

the parameter. The difficulty with this is that the parameters of interest are usually obtained by taking the first or second derivative of the data, therefore, direct reconstruction involving differentiation of noisy data can lead to large errors. Many different cosmological parameters have been suggested both these reconstruction methods (see [5] and references therein). In this work, we attempt to study the relative efficacy of the different cosmological parameters in reconstructing dark energy from observations, and discriminating between different theoretical dark energy models, using the parametric reconstruction formalism.

The paper is arranged as follows— section II contains a description of the data and methods used in the analysis, section III outlines the results, and section IV presents the conclusions.

## II. METHODOLOGY

This work attempts to classify the different cosmological parameters that characterize dark energy in terms of their efficiency in constraining the nature of dark energy. In order to do this, we analyze cosmological observations using the different parameters to compare how accurately these parameters reconstruct and discriminate between various theoretical models of dark energy. In this analysis, we primarily use Type Ia supernova data along with information on the present day matter content of the universe.

### A. Supernova Data

Type Ia supernova are the most direct evidence for the existence of dark energy at present. From early twentieth century, they were investigated as standard candles and many attempts were made to use them to measure the Hubble parameter and the deceleration of the universe [6]. The first cosmologically significant results for deceleration of the universe were produced in the late nineties, when two observational groups [7, 8] independently showed that the expansion of the universe was accelerating. Since then, there have been numerous other SNe surveys [1, 9], and despite being plagued by systematics, these remain our best observational tool for studying dark energy.

Supernova data by itself is insufficient to break the degeneracy between dark energy parameters and the curvature of the universe. Since the objective of this paper is to constrain dark energy using SNe data, we restrict our analysis to a flat model ( $\Omega_\kappa = 1$ ) of the universe which is favoured by the current CMB data [2]. The data is in the form

$$\mu_B(z) = 5\log_{10}d_L(z) + \mathcal{M}, \quad (1)$$

where  $\mathcal{M}$  represents a noise parameter usually marginalized over, and the luminosity distance  $d_L(z)$  is related to the cosmological parameters in a flat universe as–

$$d_L(z) = c(1+z) \int_0^z \frac{dz}{H(z)} \quad (2)$$

$$H(z) = \frac{\dot{a}}{a} = H_0 \sqrt{\Omega_{0m}(1+z)^3 + \Omega_{DE}(z)}. \quad (3)$$

( $a$  is the scale factor representing the expansion of the universe,  $H(z)$  is the Hubble parameter,  $c$  denotes the speed of light and  $H_0$  the present value of the Hubble parameter in km/s/Mpc).

We perform a maximum likelihood analysis on the supernova data to obtain constraints on the various dark energy parameters, the likelihood being defined as

$$\mathcal{L} \propto e^{-\chi^2/2} \quad (4)$$

$$\chi^2 = \sum_{i=1}^{N_{\text{data}}} \left( \frac{\mu_{B,i}(z_i) - \mu_B(z_i; \mathcal{M}, p_j)}{\sigma_{\mu_{B,i}}} \right)^2, \quad (5)$$

where  $p_j$  are the parameters of the fitting function chosen to represent the cosmological parameter being studied.

We use one of the most current SNe datasets [1] for our analysis. However, as we shall see, the current data is not yet of such a quality that it could strongly discriminate between different models of dark energy. We therefore also simulate three datasets based on our expectations from future surveys [10], to study the information that could be obtained from future data. The datasets used in our analysis are–

- Dataset I : Currently available Union2 dataset, with  $\sim 550$  SNe between redshifts  $z = 0 - 1.4$ , and average statistical error of  $\sigma_{\mu_B} \sim 0.1 - 0.3$  mags.

- Dataset II A : Simulated dataset based on future JDEM-like SNe surveys containing  $\sim 2000$  SNe distributed over a redshift range of  $z = 0 - 1.7$  with a larger concentration of supernovae in the midrange redshift bins ( $z = 0.4 - 1.1$ ) and average statistical errors of  $\sigma_{\mu_B} = 0.13$  mags [10]. The theoretical model used to simulate the data is the cosmological constant model with  $H_0 = 72$  km/s/Mpc, the matter density  $\Omega_{0m} = 0.27$ , and the equation of state of dark energy  $w_{DE} = -1$ .
- Dataset II B : Simulated dataset based on a JDEM-like survey as in II A, using a theoretical model of quintessence with a minimally coupled scalar field whose equation of motion is given by

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0, \quad (6)$$

with a potential [11]

$$V(\phi) = V_0\phi^\alpha; \quad \alpha = 2, \quad (7)$$

and the equation of state of dark energy

$$w_{DE} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}. \quad (8)$$

$\Omega_{0m}$  and  $H_0$  are the same as in Dataset II A.

- Dataset II C : Simulated JDEM-like dataset using a variable dark energy model with the equation of state given by [12]

$$w_{DE}(z) = w_0 + (w_m - w_0) \frac{1 + e^{\frac{1}{\Delta_t(1+z_t)}}}{1 - e^{\frac{1}{\Delta_t}}} \times \left[ 1 - \frac{e^{\frac{1}{\Delta_t}} + e^{\frac{1}{\Delta_t(1+z_t)}}}{e^{\frac{1}{\Delta_t(1+z)}} + e^{\frac{1}{\Delta_t(1+z_t)}}} \right], \quad (9)$$

with the values  $w_0 = -1.0$ ,  $w_m = -0.5$ ,  $z_t = 0.5$ ,  $\Delta_t = 0.05$ . This model has  $w_{DE} \geq -1$  everywhere, and may be realized by a standard quintessence field.  $\Omega_{0m}$  and  $H_0$  are the same as in Dataset II A.

### B. Parameters of interest

Several parameters have been suggested in the literature for reconstructing the nature of dark energy. These can be broadly divided into two categories–

- Geometrical parameters of dark energy: These are parameters that can be constructed directly from the scale factor  $a$  and its time derivatives. Examples are the Hubble parameter  $H(z)$ , and the quantity  $\Omega_m(z)$  [13], both constructed from the first

derivative of the supernova data–

$$H(z) = \frac{\dot{a}}{a}, \quad (10)$$

$$\text{Om}(z) = \frac{H^2(z)/H_0^2 - 1}{(1+z)^3 - 1}, \quad (11)$$

the deceleration parameter  $q(z)$ , constructed from the second derivative of the data–

$$q(z) = -\frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{d\log H}{d\log(1+z)}, \quad (12)$$

and the Statefinder parameters [14], constructed from the third derivative of the data–

$$r(z) = \frac{\ddot{a} a^2}{\dot{a}^3} \quad (13)$$

$$s(z) = \frac{r-1}{3(q-\frac{1}{2})}. \quad (14)$$

- Physical parameters of dark energy: These parameters, in addition to the scale factor and its derivatives, also contain physical information such as the matter density. Examples are the dark energy density normalized to the critical energy density  $\rho_{0c} = 3H_0^2/8\pi G$ , which we denote as  $\Omega_{DE}(z)$ , constructed from the first derivative of the data–

$$\Omega_{DE}(z) = \frac{H^2}{H_0^2} - \Omega_{0m}(1+z)^3, \quad (15)$$

and the equation of state of dark energy  $w_{DE}(z)$ , constructed from the second derivative of the data–

$$w_{DE}(z) = \frac{p}{\rho} \Big|_{DE} = \frac{2(1+z)/3 (d\ln H/dz) - 1}{1 - (H_0/H)^2 \Omega_{0m}(1+z)^3}. \quad (16)$$

The disadvantage for these parameters is their dependence on the physical nature of dark energy, which means that for non-physical models of dark energy which do not follow the standard Einstein equations (such as modified gravity models, or DM+DE interacting models), these parameters may not be very well-defined. Also, these parameters depend on the matter density, and therefore their reconstruction may have added bias due to uncertainty in our knowledge of the matter density.

To study the efficacy of the different parameters in quantifying dark energy, we use both geometrical and physical parameters– we select the geometrical parameters  $\text{Om}(z)$  and  $q(z)$ , and the physical parameters  $\Omega_{DE}(z)$  and  $w_{DE}(z)$ . For each parameter we need to choose a fitting function which will be used in the likelihood analysis. We experimented with different fitting functions, e.g., comparing the polynomial expansion in redshift  $z$  with that in scale factor  $a$  for  $w_{DE}(z)$ ; and in

each case we choose the fitting function which results in the best reconstruction of the parameter of interest. The fitting functions chosen for the different parameters are–

$$\text{Om}(z) = \text{Om}_0 + \text{Om}_1(1+z) \quad (17)$$

$$q(z) = q_0 + \frac{q_1 z}{1+z} \quad (18)$$

$$\begin{aligned} \Omega_{DE}(z) &= (1 - \Omega_{0m} - \Omega_1 - \Omega_2) \\ &\quad + \Omega_1(1+z) + \Omega_2(1+z)^2 \\ &= \Omega_0 + \Omega_1 z + \Omega_2 z^2 \end{aligned} \quad (19)$$

$$w_{DE}(z) = w_0 + \frac{w_1 z}{1+z}, \quad (20)$$

of which the fitting function for  $q(z)$  was first introduced in [15], that for  $\Omega_{DE}(z)$  was introduced in [14], and that for  $w_{DE}(z)$  in [16]. We use these fitting function for the likelihood estimation to obtain confidence levels on the cosmological parameters, which can then be used to discriminate between different theoretical models of dark energy.

For the geometrical parameters of dark energy, no further information is necessary for performing the analysis. However, for the physical parameters  $\Omega_{DE}(z)$  and  $w_{DE}(z)$ , information is required on the matter density. We therefore do the analysis for the physical parameters by first fixing the value of matter density to  $\Omega_{0m} = 0.27$  (which is the true value of  $\Omega_{0m}$  for the datasets II A, B, C, and is the expected value today from large scale structure), then by marginalizing over the matter density using  $\Omega_{0m} = 0.27 \pm 0.03$  [3]. The results for the second case are expected to be worse for the physical parameters, while for the geometrical parameters there is no difference since they do not depend on  $\Omega_{0m}$ .

### III. RESULTS

We first study the currently available data. The results for the Union2 dataset, i.e., dataset I, using  $\Omega_{0m} = 0.27$ , are shown in figure 1. We see that the  $2\sigma$  confidence levels for all four parameters are extremely large, and the three very different dark energy models proposed in datasets II A, B, C all fall within these confidence levels for at least a large part of the redshift range. Therefore, with the current data, at 95% confidence, three widely varying models of dark energy cannot be distinguished very well from each other using different cosmological parameters, even if the matter density were known exactly.

Since the current data cannot be used to effectively constrain theoretical models of dark energy, we now look at the constraints that may be obtained from future observations. Dataset II A is a JDEM-like simulated dataset for which the true underlying model is the cosmological constant. Figure 2 shows the  $2\sigma$  confidence levels on the different parameters for this dataset. We see that the parameter  $\text{Om}$  has the lowest errors and represents the true model quite accurately. The deceleration parameter  $q$  also has narrow errors, however, since

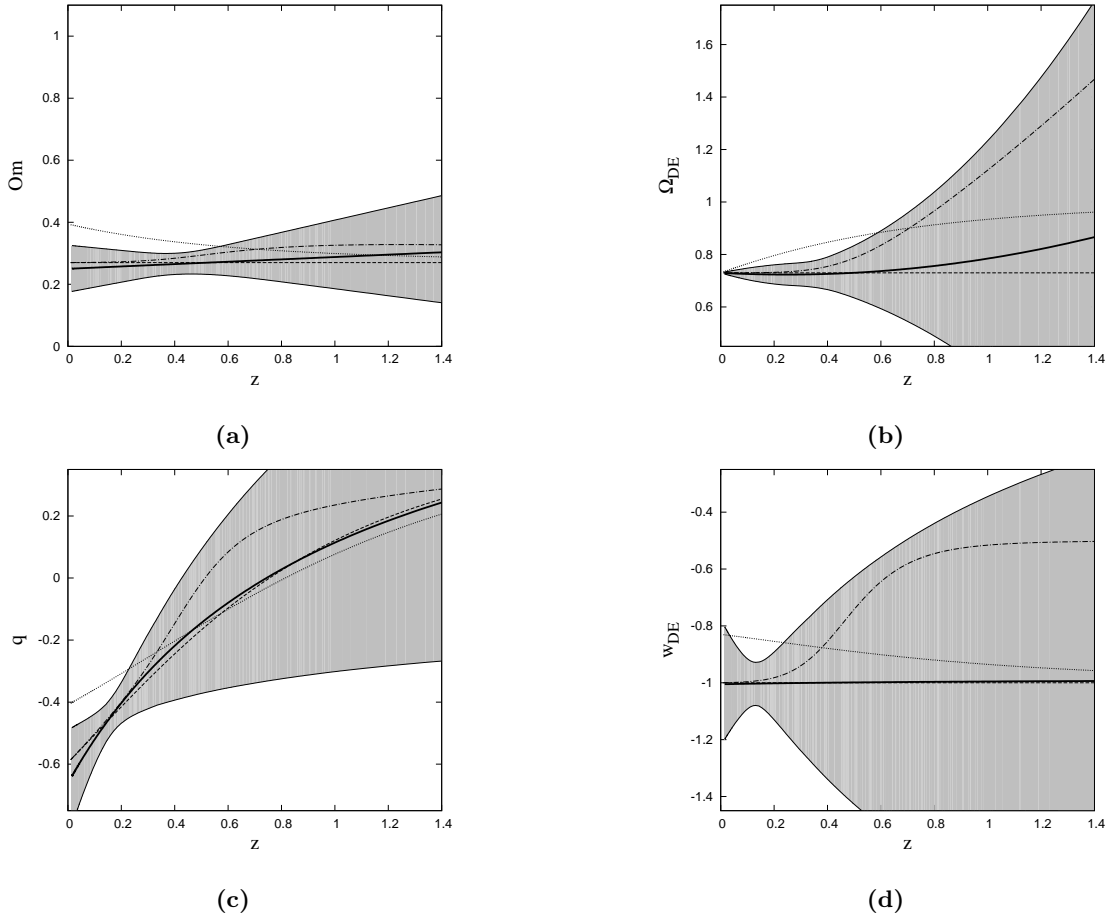


FIG. 1: Variation of different cosmological parameters with redshift for dataset I (real data). Panel (a) shows the quantity  $\Omega_m(z)$ , panel (b) shows  $\Omega_{DE}(z)$ , panel (c) shows  $q(z)$  and panel (d) shows  $w_{DE}(z)$ . The thick solid lines represent the best-fit, and the solid grey contours represent the  $2\sigma$  confidence levels. The dashed, dotted and dot-dashed lines represent the true model for datasets II A, B and C respectively. In panels (b) and (d), the solid grey contours represent the  $2\sigma$  confidence levels when  $\Omega_{0m}$  is known exactly.

the three cosmological models considered are much closer in  $q$ -space than in the other parameter spaces, even with these narrow errors,  $q$  cannot distinguish between the dark energy models of dataset II A and dataset II B. Both the equation of state  $w_{DE}$  and the energy density  $\Omega_{DE}$  have somewhat larger errors but do a better job as compared to the deceleration parameter of constraining the true model as well as distinguishing it from the other two dark energy models considered, when exact value of  $\Omega_{0m}$  is known. The parameter  $\Omega_{DE}$  has especially narrow error bars at low redshifts, but after a redshift of  $z \sim 0.7$  the error bars become larger. When value of  $\Omega_{0m}$  is not known exactly this leads to additional uncertainty in the results, as well as the possibility of bias. A higher value of  $\Omega_{0m}$  achieves a luminosity distance similar to that obtained by dark energy with an increasing  $w$ , therefore there is a degeneracy between  $\Omega_{0m}$  and the dark energy parameters, leading to the larger error bars on the parameters  $w_{DE}$  and  $\Omega_{DE}$ .

Next, we look at dataset II B, which has an underlying

model of quintessence with a slowly varying equation of state. Figure 3 shows the  $2\sigma$  confidence levels for the different parameters for this dataset. The results are very similar to that for dataset II A. The parameter  $\Omega_m$  reconstructs quite accurately with narrow error bars at high redshifts, but is slightly less effective at  $z = 0$ . The parameter  $q$  reconstructs accurately at low  $z$  at less so at high  $z$ . The parameter  $\Omega_{DE}$  is excellent at reconstruction at low redshift when  $\Omega_{0m}$  is known, while  $w$  reconstructs reasonably as well when  $\Omega_{0m}$  is known. When  $\Omega_{0m}$  is not known accurately, the reconstruction has a bias for  $\Omega_{DE}$  and  $w$ , and the error bars are larger.

Dataset II C uses a model with a stronger variation of the equation of state, from  $w_{DE} = -1$  today to  $w_{DE} = -0.5$  at  $z = 1$ . The results for this dataset are shown in figure 4. We see that both the deceleration parameter  $q$  and the equation of state  $w_{DE}$  are unable to reconstruct the true model (dot-dashed lines) accurately, because the parameterization chosen is not general enough to reconstruct the true model accurately.

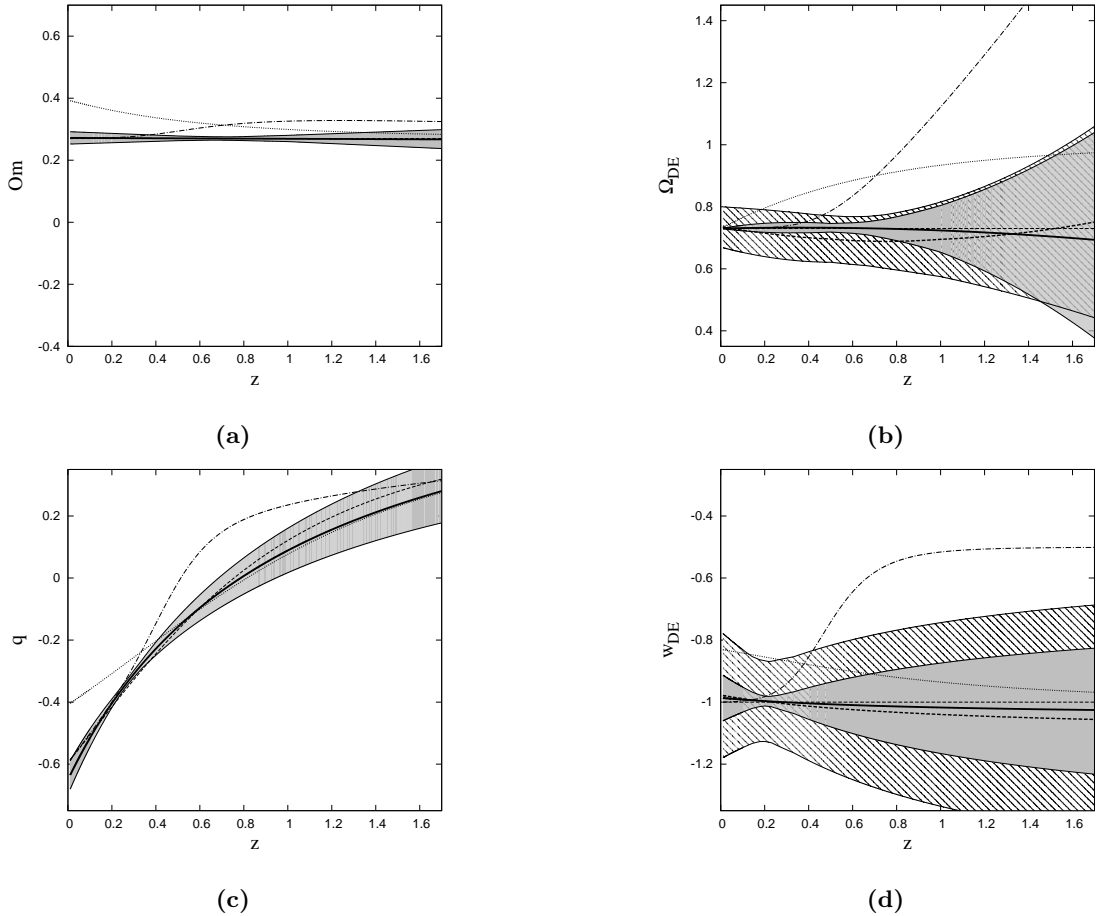


FIG. 2: Variation of different cosmological parameters with redshift for dataset II A. Panel (a) shows the quantity  $Om(z)$ , panel (b) shows  $\Omega_{DE}(z)$ , panel (c) shows  $q(z)$  and panel (d) shows  $w_{DE}(z)$ . The shaded contours represent the  $2\sigma$  confidence levels, the thick solid lines represent the best-fit, the thick dashed lines in panels (b) and (d) represent the best-fit when  $\Omega_{0m}$  is marginalized over. The dashed, dotted and dot-dashed lines represent the true model for datasets II A, B and C respectively. In panels (b) and (d), the grey solid contours represent the  $2\sigma$  confidence levels when  $\Omega_{0m}$  is known exactly, while the hatched contours are marginalized over  $\Omega_{0m} = 0.27 \pm 0.03$ .

Even when  $\Omega_{0m}$  is known, the equation of state appears to favour  $w_{DE} < -1$  at  $z = 0$  at  $2\sigma$ , which would lead to an extremely different model of dark energy violating the Weak Energy Condition. The dark energy density  $\Omega_{DE}$  does better in reconstructing the true model when  $\Omega_{0m}$  is known, especially at low redshift. This is because the dark energy density, which is obtained from the first derivative of the SNe data, has usually less sharp variations than the equation of state or deceleration parameter, which are related to the second derivative of the data. However, when  $\Omega_{0m}$  is not known exactly,  $\Omega_{DE}$  performs much worse, especially at low redshift, because of the degeneracy between  $\Omega_{DE}$  and the matter density. As in the previous cases,  $Om$  appears to capture the behaviour of this model very well.

As we see from the figures 2, 3, 4, the different parameters perform very differently in reconstructing the models chosen. We quantify the efficiency of the different parameters using a distance criterion which calculates

the distance of the true model from the best-fit, weighted by the  $2\sigma$  errors obtained by the fitting procedure. The lower this value, the more accurate the reconstruction, and the narrower the confidence levels on the parameter. A large value of this parameter could mean either very large errors, which lowers the discriminatory power of the parameter, or a biased fit that is far away from the true result. We also quantify the distance criterion for the other two models in each case and take the ratio of this quantity for the true model to that for the nearest of the other models, since this highlights the discriminatory power of the parameter. The lower this value, the better the parameter at discriminating other models from the true model. We note that this quantity is of course dependent on the models chosen, however, for the three models studied here, it can give a comparative assessment of the discriminatory power of the parameters considered. For a different set of models, the value of this quantity would obviously change, but the compari-

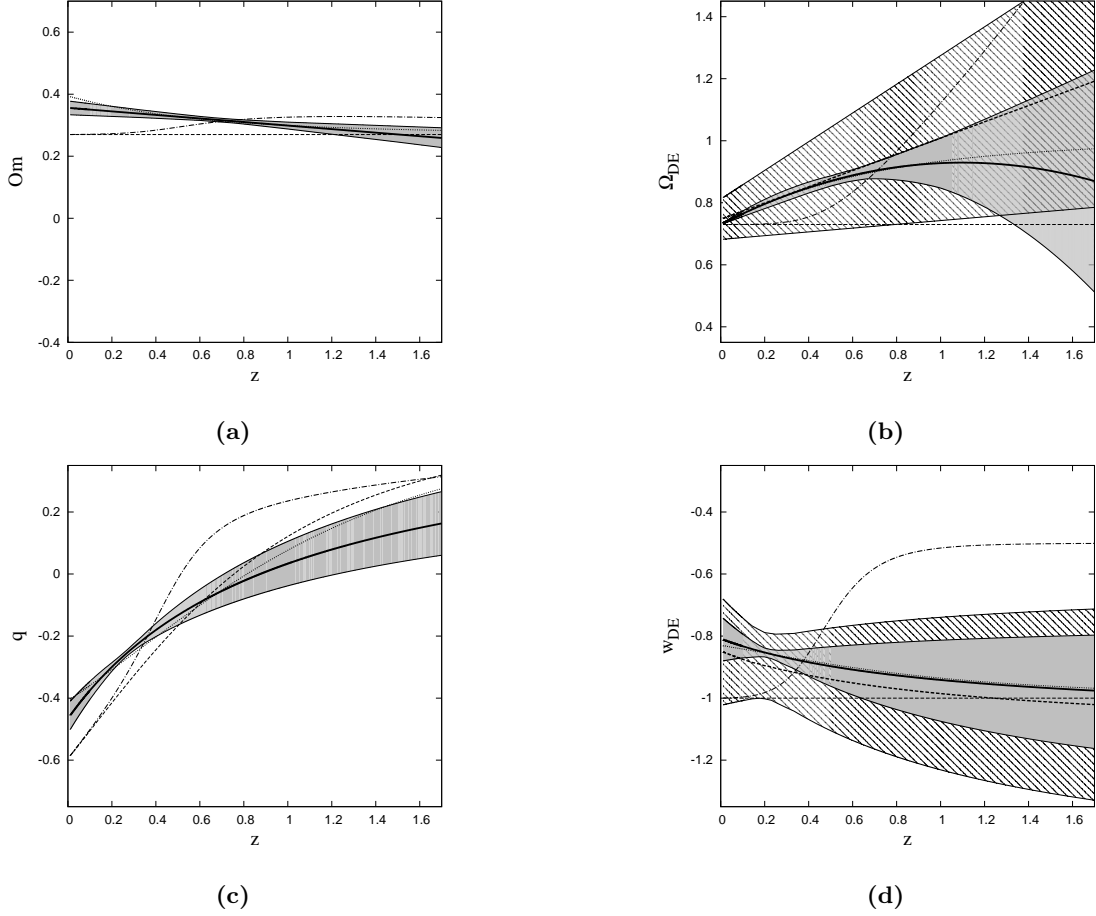


FIG. 3: Variation of different cosmological parameters with redshift for dataset II B. Panel (a) shows the quantity  $\Omega_m(z)$ , panel (b) shows  $\Omega_{DE}(z)$ , panel (c) shows  $q(z)$  and panel (d) shows  $w_{DE}(z)$ . The shaded contours represent the  $2\sigma$  confidence levels, the thick solid lines represent the best-fit, the thick dashed lines in panels (b) and (d) represent the best-fit when  $\Omega_{0m}$  is marginalized over. The dashed, dotted and dot-dashed lines represent the true model for datasets II A, B and C respectively. In panels (b) and (d), the grey solid contours represent the  $2\sigma$  confidence levels when  $\Omega_{0m}$  is known exactly, while the hatched contours are marginalized over  $\Omega_{0m} = 0.27 \pm 0.03$ .

son between the cosmological parameters would be similar, especially since the three models chosen are quite different from each other in behaviour. For a parameter  $p$ , the efficiency criterion is defined as—

$$\delta_p^2 = \sum_i (p_{i,\text{true}} - p_{i,\text{fit}})^2 \sigma_{p(i,\text{fit})}^2, \quad (21)$$

and the discrimination ratio is defined as—

$$\frac{\delta_p^2(\text{True Model})}{\delta_p^2(\text{Nearest Model})}, \quad (22)$$

where  $\sigma_{p(i,\text{fit})}^2$  is the  $2\sigma$  error on the reconstructed parameter.

The above quantities are calculated in Table I for the different parameters for all three models. We see that for the different parameters, the  $\chi^2$  on the data is quite similar for all cases, showing that they all reconstruct successfully in the data space. However, since the parameter

space is linked to the data space by one or more differentiations, the reconstruction of the actual parameters, quantified by  $\delta_p^2$  (21), is quite different for the different parameters. The quantity  $\Omega_m$  performs consistently well in reconstructing the dark energy models considered, and has the lowest values of  $\delta_p^2$ . The deceleration parameter  $q$  performs well for Models A, B, but does not do so well for Model C which is a model not well characterized by the parameterization chosen for it. The dark energy density and the dark energy equation of state both work reasonably when the matter density is known, however, their performance degenerates when uncertainty is introduced in  $\Omega_{0m}$ , especially in Model C which is the most strongly varying. The discrimination criterion (last column of table) shows how well the parameter can discriminate the true model from the other two models considered. We see that the parameter  $\Omega_m$  performs very well on this criterion as well. The deceleration parameter, which performed reasonably well on  $\delta_p^2$ , does not perform as well on

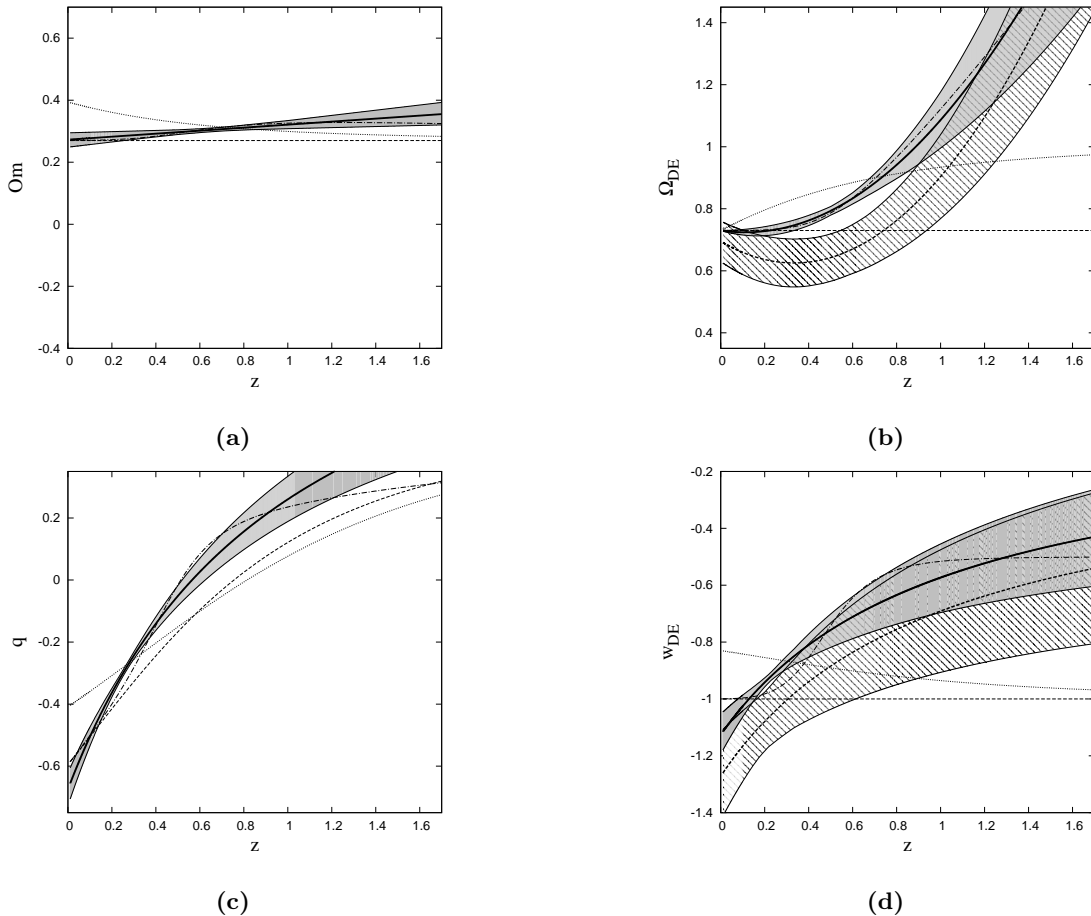


FIG. 4: Variation of different cosmological parameters with redshift for dataset II C. Panel (a) shows the quantity  $Om(z)$ , panel (b) shows  $\Omega_{DE}(z)$ , panel (c) shows  $q(z)$  and panel (d) shows  $w_{DE}(z)$ . The shaded contours represent the  $2\sigma$  confidence levels, the thick solid lines represent the best-fit, the thick dashed lines in panels (b) and (d) represent the best-fit when  $\Omega_{0m}$  is marginalized over. The dashed, dotted and dot-dashed lines represent the true model for datasets II A, B and C respectively. In panels (b) and (d), the grey solid contours represent the  $2\sigma$  confidence levels when  $\Omega_{0m}$  is known exactly, while the hatched contours are marginalized over  $\Omega_{0m} = 0.27 \pm 0.03$ .

this criterion, because the different cosmological models are quite close together when represented by this quantity. Thus the deceleration parameter is not very useful when it comes to choosing one dark energy model over the others. The dark energy density  $\Omega_{DE}$  and the equation of state  $w_{DE}$  both perform reasonably when  $\Omega_{0m}$  is known exactly, but once again their performance degenerates when priors are put on  $\Omega_{0m}$ , since the error bars increase. Overall, in both the criteria considered,  $Om$  appears to perform the best, while  $\Omega_{DE}$  performs well at low  $z$  provided  $\Omega_{0m}$  is known, and  $w$  performs moderately well when  $\Omega_{0m}$  is known.

#### IV. CONCLUSIONS

We reconstruct four different cosmological parameters to examine their potential in extracting information from observations about dark energy. We find that the pa-

rameter  $Om$ , which is constructed from the Hubble parameter, has the narrowest errors, and gives the most accurate reconstruction. We also see that results for the physical parameters  $\Omega_{DE}$ ,  $w_{DE}$  are extremely dependent on knowledge of the matter density, and without very tight bounds on matter density, physical parameters of dark energy may not perform well in reconstruction dark energy. The deceleration parameter reconstructs quite well, but has low discriminatory powers, since different models of dark energy have very similar values for this parameter.

Overall it appears that the parameters constructed out of the first derivative of the data, i.e. the Hubble parameter, are somewhat better poised to give information about the nature of dark energy than the second derivatives, and geometric parameters of dark energy, such as  $Om$ , are better at reconstructing dark energy since they are not biased by lack of knowledge about the matter density, provided they are constructed in such a way that

TABLE I:  $\chi^2_{\min}$  for the different parameter reconstructions using the three simulated datasets, the efficiency criterion (21) which defines how well the model reconstructs the truth, and the discrimination criterion (22), which defines how well the model discriminates from the other two truths considered.

Dataset	$\Omega_{0m}$	Parameterization	$\chi^2_{\min}$	$\delta_p^2 = \sum (p_{\text{true}} - p_{\text{fit}})^2 \sigma_{p(\text{fit})}^2$	$\frac{\delta_p^2(\text{True Model})}{\delta_p^2(\text{Nearest Model})}$
II A	0.27	$\text{Om}(z) = \text{Om}_0 + \text{Om}_1(1+z)$	2349.48	$10^{-5}$	0.001
		$q(z) = q_0 - q_1 z/(1+z)$	2349.77	0.011	0.668
		$\Omega_{DE}(z) = \Omega_0 + \Omega_1 z + \Omega_2 z^2$	2349.45	0.019	0.009
		$w(z) = w_0 - w_1 z/(1+z)$	2349.44	0.014	0.010
	$0.27 \pm 0.03$	$\Omega_{DE}(z) = \Omega_0 + \Omega_1 z + \Omega_2 z^2$	2349.03	0.164	0.882
		$w(z) = w_0 - w_1 z/(1+z)$	2349.26	0.173	1.395
II B	0.27	$\text{Om}(z) = \text{Om}_0 + \text{Om}_1(1+z)$	2349.75	0.0002	0.007
		$q(z) = q_0 - q_1 z/(1+z)$	2350.08	0.047	0.652
		$\Omega_{DE}(z) = \Omega_0 + \Omega_1 z + \Omega_2 z^2$	2349.54	0.119	0.010
		$w(z) = w_0 - w_1 z/(1+z)$	2349.45	0.042	0.009
	$0.27 \pm 0.03$	$\Omega_{DE}(z) = \Omega_0 + \Omega_1 z + \Omega_2 z^2$	2349.18	0.292	1.002
		$w(z) = w_0 - w_1 z/(1+z)$	2349.37	0.201	1.292
II C	0.27	$\text{Om}(z) = \text{Om}_0 + \text{Om}_1(1+z)$	2349.75	0.0001	0.006
		$q(z) = q_0 - q_1 z/(1+z)$	2349.50	0.104	0.479
		$\Omega_{DE}(z) = \Omega_0 + \Omega_1 z + \Omega_2 z^2$	2349.46	0.039	0.007
		$w(z) = w_0 - w_1 z/(1+z)$	2349.97	0.077	0.023
	$0.27 \pm 0.03$	$\Omega_{DE}(z) = \Omega_0 + \Omega_1 z + \Omega_2 z^2$	2349.19	1.981	0.452
		$w(z) = w_0 - w_1 z/(1+z)$	2348.96	1.163	0.356

they can discriminate between different models of dark energy. To obtain information about dark energy from the physical parameters of dark energy, it is important to have independent sources of observation of the matter density. Thus, for a parameter to obtain maximum information out of the data, it works better if it is constructed out of the first derivative of the data or less, does not depend on other physical parameters such as the matter density, and is constructed such that different dark energy models can be easily discriminated from each other with the parameter.

We note here that the reconstruction methods considered here are that of parametric reconstruction, non-parametric reconstruction of cosmological parameters would suffer less from biases in reconstruction such as those Model C. However, typically these methods have higher errors, and also the issues of degeneracy with

other, non-dark energy parameters exist for these methods as well. Therefore if a simple cosmological parameter, like  $\text{Om}$ , can be constructed which performs well for a large class of models, both in reconstructing and in discriminating between models, and is also independent of other non-dark energy parameters such as  $\Omega_{0m}$ , it would be extremely useful for understanding the nature of dark energy.

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